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Lifetime distribution of spontaneous emission from line antennas in two-dimensional quasi-periodic photonic crystals

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Abstract
We investigate the lifetime distribution functions of spontaneous emission from line antennas embedded in finite-size two-dimensional 12-fold quasi-periodic photonic crystals. Our calculations indicate that two-dimensional quasi-periodic crystals lead to the coexistence of both accelerated and inhibited decay processes. The decay behaviors of line antennas are drastically changed as the locations of the antennas are varied from the center to the edge in quasi-periodic photonic crystals and the location of transition frequency is varied.

1. Introduction

Photonic band gap materials have been demonstrated to have potential applications in many fields. For example, photonic crystals (PCs) can suppress vacuum fluctuations and spontaneous emission (SE) [1, 2] and can cause novel quantum electrodynamic effects [3]. Controlling SE using PCs is a topic of fundamental importance [4]. The SE from atoms in a PC differs greatly from those in uniform material, e.g., they cannot simply be described using a single lifetime due to their wide lifetime distribution [5–7]. Studies on the lifetimes of SE from atoms in two- and three-dimensional PCs [8–10] have demonstrated that the SE could effectively be controlled by PCs, and that accelerated and inhibited decay processes coexisted. Previous theoretical studies on lifetime distributions have mainly focused on periodic PCs of infinite size [8, 9]. However, most experimental research has focused on two-dimensional finite-size PCs. Some interesting results, on the other hand, have been found in quasi-periodic photonic crystals (QPCs) [11, 12]. These results have differed from those on periodic crystals. We speculated whether quasi-periodicity could effectively control SE and whether it would have new features in quasi-periodic crystals. In this study, we used finite-size QPCs to control the SE of line antennas. We combined Green's function with a method of multiple scattering [13] to calculate the local density of states (LDOS) in QPCs, and then calculated the lifetime distribution function (LDF) of line antennas in two-dimensional QPCs of finite size.

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2. Theory and structure

To understand how SE is controlled by two-dimensional QPCs, the LDF in PCs should be investigated. The LDF reflects the fact that an assembly of emitters in the material has a distribution of emission lifetimes, rather than only single lifetime decay. The emission intensity from these emitters with these lifetimes will be proportional to the LDF [7]. The LDF

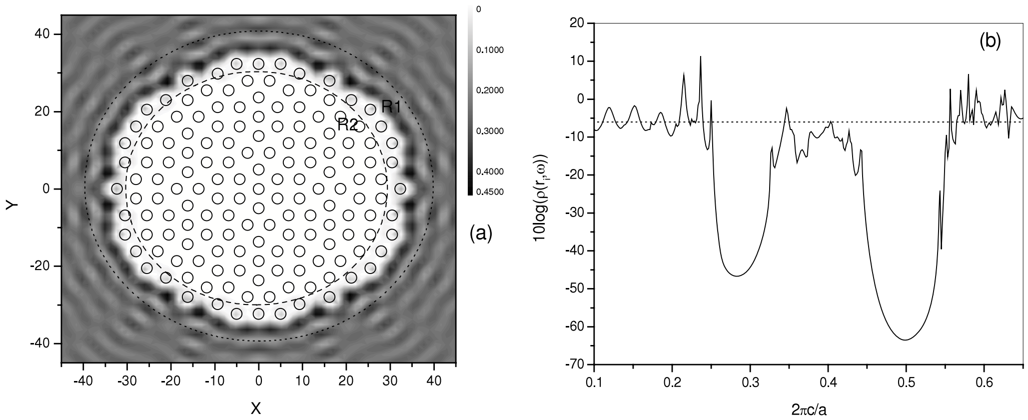

Figure 1. (a) Schematic diagram of a two-dimensional amorphous PC; the radius of circle R1 (dotted line) is $8a$ and the radius of circle R2 (dashed line) is $6a$. Two circles delineate three areas we studied: the entire crystal area is within circle R1, the center area is within circle R2, and the edge area is between circles R1 and R2. The distributed LDOS is at the frequency in the first gap center. (b) LDOS at the center of the quasi-crystal versus frequencies of TM polarized light. The dotted line is the LDOS in vacuum.

is defined [7, 8] as

$$D(\tilde{\tau}, \omega) = \sum_i W_i \frac{1}{\sqrt{\pi}\sigma} \exp\left[\frac{-\left[\tilde{\tau} - \tilde{\tau}(\mathbf{r}_i, \omega)\right]^2}{\sigma^2}\right], \quad (1)$$

where $\mathbf{r}_i$ denotes the position of the ith emitter and W_i is a weight factor. When line antennas are homogeneously distributed in space, $W_i = 1$. Tunable parameter σ is used to smooth the LDF curve. A σ of 0.05 was adopted to guarantee a smooth curve in this paper. The optimal σ value of 0.05 was determined using the resulting LDF curve. This ensured that the main distribution of lifetime would not be affected by random isolated peaks in the calculated LDF curves. Moreover, $\tilde{\tau}(\mathbf{r}_i, \omega) = \tau(\mathbf{r}_i, \omega)/\tau_f(\omega)$, where $\tau(\mathbf{r}_i, \omega)$ and $\tau_f(\omega)$ are the SE lifetimes at a given position, $\mathbf{r}_i$, in a PC and in a homogeneous medium. The summation covers all the line antennas distributed in the area over which LDF is calculated. $\tau(\mathbf{r}_i, \omega) = 1/\Gamma(\mathbf{r}_i, \omega)$, where $\Gamma(\mathbf{r}_i, \omega)$ is the transition probability of the line antennas, which can be expressed by the LDOS, $\rho(\mathbf{r}_i, \omega)$ [14]:

$$\Gamma(\mathbf{r}_i, \omega) = 2\pi\rho(\mathbf{r}_i, \omega). \quad (2)$$

Several methods have been used to calculate the density of states (DOS) and LDOS in PCs [15, 16]. However, the LDOS or DOS in finite-size PCs have not been studied much. Asatryan *et al* extended the exact formalism of multiple expansions to construct a two-dimensional Green's function for finite-size two-dimensional PCs [17]. The photon Green's function is an important tool for exploiting photon behavior. An appropriate Green's function can be used to analyze the underlying physics of SE controlled by PCs [18]. In our calculations, the source was a line antenna of infinite length parallel to the cylinder axes. The LDOS in the sample can be written as

$$\rho(\mathbf{r}_i; \omega) = -\frac{2\omega}{\pi c^2} \mathrm{Im}\left[G(\mathbf{r}_i, \mathbf{r}_s; \omega)\right], \quad (3)$$

where $G(r_i, r_s, \omega)$ is the electromagnetic Green's function with an antenna at position $\mathbf{r}_s$ and an observation point at $\mathbf{r}_i$ [17]. Here, ω is the angular frequency of an electromagnetic wave, and c is the light velocity in a vacuum. In the following calculation, we have only considered TM polarization, i.e. $G = E_z$, where the magnetic fields are polarized parallel to the cylinder axes. We assumed that 10 201 line antennas (only the line antennas used in Green's function) were distributed homogeneously throughout an area of $26a \times 26a$ where LDOS was calculated, where a is the unit of length. The calculated LDOS revealed how the PCs affected the radiation properties of antennas embedded in them.

A two-dimensional 12-fold QPC is constructed using 157 cylindrical dielectric rods each with a dielectric constant of 8.4. The rods are placed at each vertex of a dodecagonal quasi-periodic pattern and embedded in a styrofoam template with a dielectric constant of 1.04 (dielectric constant of styrofoam). The resulting 12-fold quasi-crystal pattern tiled by squares and triangles is shown in figure 1(a), which has a similar structure to those reported elsewhere [12, 19]. The squares and the triangles have the same side length a, and the radius of the cylinder is $0.34a$. Using equation (3) and multiple scattering, we calculated the LDOS in the QPC. The distribution of the LDOS in a quasi-crystal at a frequency of $0.505(2\pi c/a)$ at the center of the photonic band gap is shown in figure 1(a), which has a similar structure to those reported elsewhere [12, 19]. The results in this figure indicate that the LDOS is very small throughout the PC except at its edge. Figure 1(b) shows the LDOS at the center of the PC versus the frequencies of TM polarized light. Two intervals, representing the photonic band gaps, which range from $0.253(2\pi c/a)$ to $0.325(2\pi c/a)$ and from $0.446(2\pi c/a)$ to $0546(2\pi c/a)$, can be seen in the spectrum.

3. Lifetime distribution functions in PCs

We know that the SE in the sample is determined by LDOS. Using LDOS and equation (1), we calculated the LDF of line antennas at different frequencies in finite-size crystals. To find the pure quasi-periodic crystal effect, a reference lifetime, τ_f, was chosen using the lifetime of line antennas embedded in a

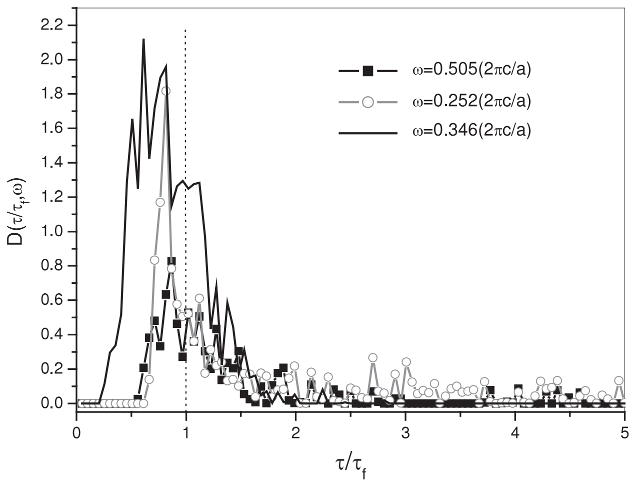

Figure 2. LDFs in entire PC area of two-dimensional quasi-periodic PC consisting of dielectric rods. Transition frequencies are set at $0.505(2\pi c/a)$ in the gap, $0.252(2\pi c/a)$ near the photonic band edge, and $0.346(2\pi c/a)$ in the photonic band and are indicated by three curves in this figure. The unit of frequency ω in figure is $(2\pi c/a)$.

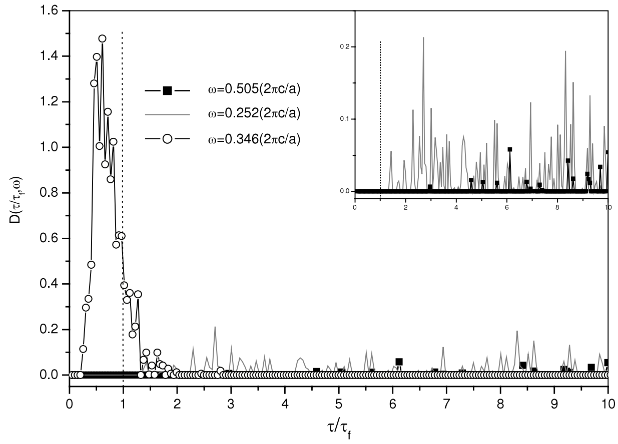

Figure 3. LDFs in the central area (inside circle R2) in a two-dimensional 12-fold QPC consisting of dielectric rods with transition frequencies of $0.505(2\pi c/a)$ in the photonic gap, $0.252(2\pi c/a)$ near the photonic band edge, and $0.346(2\pi c/a)$ in the photonic band. Inset: comparison of LDFs for a transition frequency of $0.252(2\pi c/a)$ (square line) and a transition frequency of $0.346(2\pi c/a)$ (solid gray line).

homogeneous background medium with a dielectric constant, ε, of 1.04, which is indicated by the vertical dotted line located at $\tilde{\tau} = 1$ in all the figures that follow. If lifetimes are distributed within a range of $\tilde{\tau} < 1$, this means that, compared with those in homogeneous material, most SE from line antennas is accelerated; otherwise it is inhibited. We examined three different areas in finite-size QPCs. The first, called the entire PC area, was inside a circle, R1, with a radius of $8a$. The second, called the center area, was the area inside circle R2 with a radius of $6a$. The third area, called the edge area, was that between circles R1 and R2. This division was useful for us to study the SE from antennas in different areas. The center area is equivalent to the unit cell for an ideal model of an infinite-size PC [6, 8], which does not take into account the effect of the PC edge. The edge area, in contrast, was studied to determine what effect the PC edge had on the SE. As a result, we could study the lifetime distribution within different areas in a PC of finite size.

Figure 2 plots the LDFs of line antennas within the entire PC area. The results indicate that the entire (whole) quasi-crystal effect resulted in both an accelerated and an inhibited component coexisting in spontaneous decay kinetics. The results also indicate that the majority of lifetimes are distributed within the range of $\tilde{\tau} < 1$ at frequencies of $0.252(2\pi c/a)$ and $0.346(2\pi c/a)$, where the acceleration components are about 80% of the total lifetimes. This means that the SE is accelerated for the majority of line antennas with transition frequencies near the photonic band edge and in the photonic band. At a transition frequency of $0.505(2\pi c/a)$ in the center of the band gap, about half the lifetimes are distributed within the range of $\tilde{\tau} < 1$, which means that about half the SE of the line antennas is inhibited and the other half is accelerated.

Figure 3 plots the calculated LDFs of the line antennas in the center area. The other parameters are the same as those used in figure 2. Investigating the SE in this center area can minimize the effects of the surface of the crystal. We found that the accelerated components became larger at a frequency of $0.346(2\pi c/a)$ in the photonic band compared with that in figure 2. This suggests that the SE of line antennas in the center area with a transition frequency in the photonic band will be significantly enhanced. Other differences should be noted; the lifetime distribution for frequencies of $0.252(2\pi c/a)$ and $0.505(2\pi c/a)$ completely shifts to the $\tilde{\tau} > 1$ range, where the SE of line antennas is totally inhibited. The lifetime distribution for a frequency of $0.505(2\pi c/a)$ is farther from line $\tilde{\tau} = 1$ than that in figure 2. Moreover, there are some peaks within the range of $\tilde{\tau} > 1$ in the LDF curves in figure 3. These results indicate that the SE in the QPCs drastically decreases at the transition frequencies in the photonic band gap and near the photonic band edge, and that the SE of the line antennas in the QPC occurs in isolated positions. These behaviors are due to the small LDOS in the central area of the crystal (inside circle R2) at frequencies in the photonic band gap and near the photonic band edge.

Figure 4 plots two curves at a frequency of $0.346(2\pi c/a)$ in the photonic band and a frequency of $0.505(2\pi c/a)$ in the photonic band gap in the edge area (area between circles R1 and R2). Both the curves are distributed either side of $\tilde{\tau} = 1$. The acceleration and inhibition processes coexist, which means that the lifetimes of some line antennas are decreased and others are increased for both transition frequencies. Figure 5 compares the LDFs in the edge (solid line) and central areas (circle line) at a transition frequency of $0.252(2\pi c/a)$ near the photonic band edge. The acceleration component of line antennas in the edge area increases, while the inhibition component in the center area increases at a frequency of $0.252(2\pi c/a)$ compared with that in the entire PC area in figure 2. If we move the antennas from the center to the edge area of the crystal, the SE changes from being inhibited to being accelerated. Why is the SE of the majority of antennas

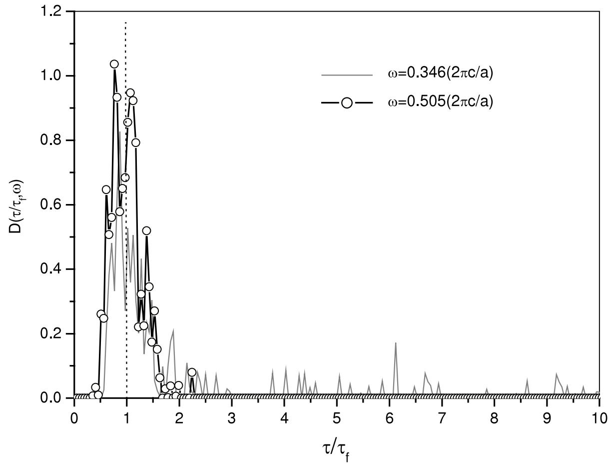

Figure 4. LDFs in the edge area (area between circles R1 and R2) in a two-dimensional 12-fold QPC consisting of dielectric rods with transition frequencies of $0.505(2\pi c/a)$ in the photonic band gap and $0.346(2\pi c/a)$ in the photonic band.

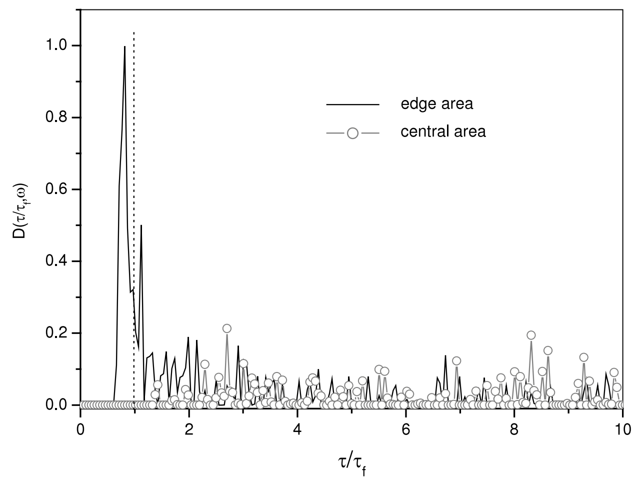

Figure 5. Comparison of LDFs in the edge area (solid line) and central area (circle line) with a transition frequency of $0.252(2\pi c/a)$ near the photonic band edge.

enhanced in the edge area but completely inhibited in the central area with a frequency near the photonic band edge? We think the reason is related to the finite size of the QPCs. Figure 1(a) shows the calculated LDOS of the crystal at a frequency in the band gap, where the quasi-resonant modes due to the cluster of cylinders can be found near the edge of the crystal. The quasi-resonant mode is the one that is deformed by the quasi-periodic and irregular structure of the edge of the QPC as it is near the edge of the quasi-periodic photonic crystal. We also found that there is a quasi-resonant mode in the boundary (edge) of the sample at a frequency of $0.505(2\pi c/a)$ in the photonic band gap and a frequency of $0.252(2\pi c/a)$ near the band gap edge, while there are no quasi-resonant modes at a frequency of $0.346(2\pi c/a)$ in the photonic band. The existence of quasi-resonant modes can directly change the number of SEs at the sample edge. Therefore, we deduced that the enhanced SE of line antennas in the edge area at transition frequencies in the gap and near the band edge was mainly due to the finite size of the crystal.

The large acceleration component of emission from line antennas in the edge area results from the finite-size effect, and the enhanced emission is due to interference between the QPC and vacuum in the edge region (here, the styrofoam in the background acted as an approximate vacuum). Here, let us explain the influence of the size of the computational cell on the LDFs. Asatryan *et al* studied the LDOS in periodic PCs of different sizes [17]. They found that, like the frequency in the photonic band gap for a given cluster size (PC size), the LDOS decreases exponentially towards the center, it is small everywhere in the interior of the structure, and there is a boundary layer with a thickness of roughly a single lattice constant that separates the cluster's interior from the vacuum [17]. Moreover, we can determine LDOS from the results calculated by Asatryan *et al* [17], i.e. those near the edge for different cluster sizes. Even though the positions of the minima and maxima change with size, their amplitudes undergo no large changes. Therefore, we think the effect of interference causes no large changes between the periodic PC and free space as the size of the PC changes. We also adapted this finding to QPC. We calculated the LDOS and LDF for QPCs of different sizes, and we found that the shape of the LDF curve, the peak position on the time axis, and the degrees of enhancement and inhibition underwent no notable changes as the QPCs changed in size. However, the LDF's amplitude increased as the QPCs increased in size. This was because the edge area increased as the PC size increased, and the total light field and LDOS consequently increased around the edge area.

4. Lifetime distributions in PCs

Figures 6(a)–(c) show variations in lifetimes at the antenna position in a quasi-periodic crystal with respective frequencies of $0.505(2\pi c/a)$ in the band gap, $0.252(2\pi c/a)$ near the band gap edge, and $0.346(2\pi c/a)$ in the photonic band. The lifetimes differ significantly at different frequencies. The longest lifetime at a frequency of $0.505(2\pi c/a)$ in the photonic gap is 3.39×10^6 (au), and the average lifetime is 1.21×10^4 (au) at a frequency of $0.505(2\pi c/a)$ in figure 6(a). A long lifetime indicates that the emission has an inhibitive effect in the photonic gap. The relative width of the lifetime distribution (RWOLD), $T_{\rm rw} = (\tau_{\max} - \tau_{\min})/\tau_{\min}$, is as high as 9.44×10^6 (au) at the frequency of $0.505(2\pi c/a)$ in the photonic gap. The RWOLD at the frequency of $0.252(2\pi c/a)$ near the band gap edge is 1275, and the average lifetime is 5.81. The RWOLD at the frequency of $0.346(2\pi c/a)$ in the photonic band is 9.8, and the average lifetime is 0.61. The reference lifetime in the vacuum is 0.64 ($1/(2\pi \times 0.25)$, where 0.25 is the LDOS in a vacuum). The inhibitive effect weakens as the transition frequency shifts from the photonic band gap to the photonic band edge and the photonic band. The lifetime at the center of the crystal is larger than that in the edge area in the photonic gap and near the band edge. Looking at the lifetime distribution at the frequency near the band edge in figure 6(b), the lifetime in the high-index regions is smaller than that in the low-index ones. The lifetime distribution at

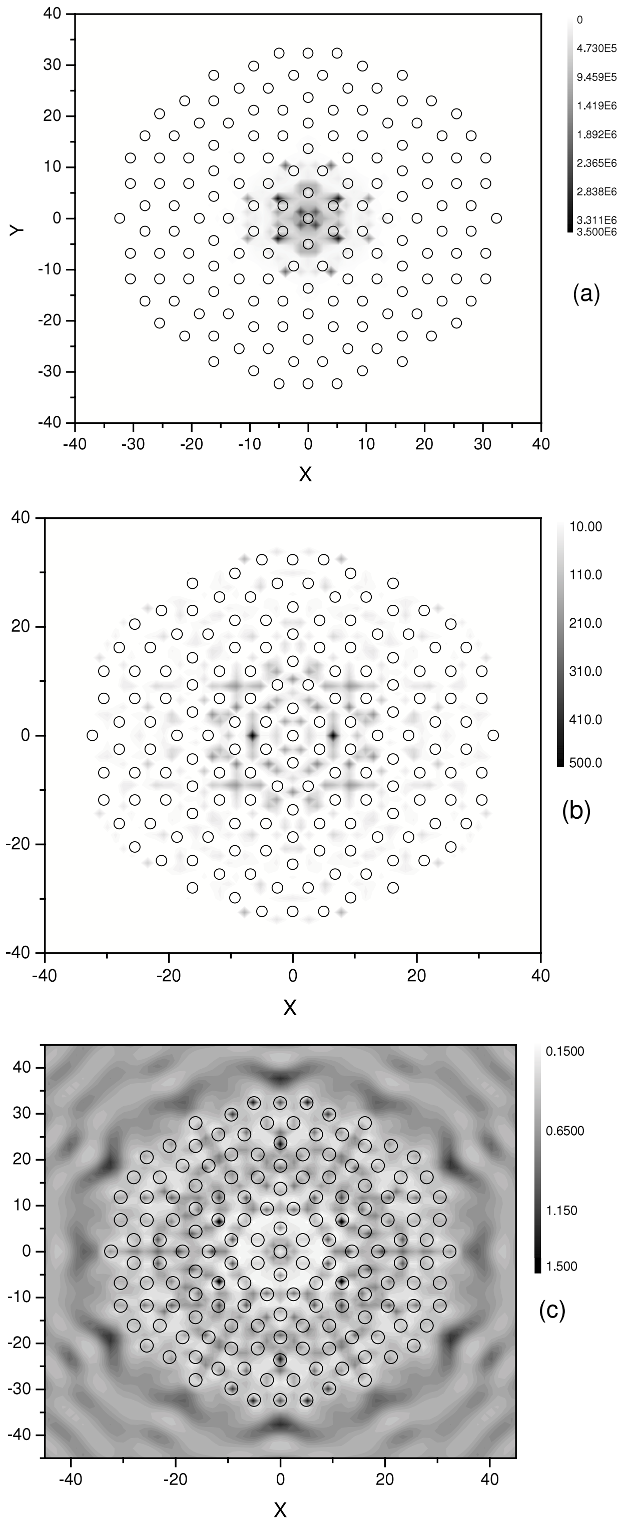

Figure 6. Lifetime distributions as function of position at frequencies of (a) $0.505(2\pi c/a)$ in the photonic band gap, (b) $0.252(2\pi c/a)$ near the photonic band edge, and (c) $0.346(2\pi c/a)$ in the photonic band.

a frequency of $0.346(2\pi c/a)$ in the photonic band is shown in figure 6(c), which has a symmetric quasi-periodic pattern. The distribution of lifetimes is determined by the structure of quasi-periodic crystals. QPCs have neither true periodicity nor translation symmetry but do have a quasi-periodicity that exhibits long-range order and orientation symmetry. These enable QPCs to contain many inequivalent sites, which have a more significant effect on SE than those in a periodic system. Notomi *et al* demonstrated that lasing properties could be controlled using quasi-periodicity and that lasing action was due to feedback on the quasi-periodic long-range order [20]. We can also see that the lifetimes of SE depend on the positions of the line antennas and we believe that the lifetime distribution is determined by the quasi-periodic long-range order. These lifetime distributions differ from those in a periodic structure [6, 21]. We previously studied [21] the LDFs in PC for different degrees of disorder with frequencies in the photonic band, band edge, and band gap. We found that the LDF curves did not change significantly when the degree of disorder varied. For all three frequencies, some sharp peaks appeared in the LDF curves for periodic PC, while the LDF curves for disordered PC were smoother. This is because LDOS is periodically distributed within the periodic structure, and homogeneously distributed within the disordered structure. The LDOS is quasi-periodically distributed in a QPC. Therefore, we found the LDF curves for a QPC were not as smooth as those for a disordered PC.

We found that there were enhanced effects on SE at a frequency of $0.252(2\pi c/a)$ near the band edge (figures 2 and 5) and a frequency of $0.346(2\pi c/a)$ in the photonic band (figures 2 and 3), but their mechanisms differed. The enhanced emission of line antennas at a frequency of $0.346(2\pi c/a)$ in the photonic band resulted from the large LDOS in the QPC. Correspondingly, the lifetime of SE is short, as shown in figure 6(c). However, the enhanced emission at a frequency near the band edge is due to the large field of the quasi-resonant modes of the finite-size QPC composed of cylinders. We can find the dependence of SE on the frequency location of the photonic band gap and the position of the line antennas in the QPC. Therefore, we can control the SE by varying the atomic position and the parameters of the quasi-periodic crystal. We can fabricate 12-fold QPCs with different parameters to control the emission of line antennas using a defined emission frequency. If we want to enhance emission, we can design a photonic band gap edge or an appropriate photonic band located at the emission frequency. However, if we want to inhibit emission, we can design a photonic band gap center located at the emission frequency and place line antennas in the central area of the QPC.

The method we used can also be extended to three-dimensional PCs. To do this, we need to calculate the three-dimensional LDOS in three-dimensional PCs. The three-dimensional LDOS can be defined for dipole point-like atoms from Green's functions. The three-dimensional LDOS have already been calculated in a finite two-dimensional PC using three-dimensional multiple scattering [22]. A similar method called the layer Korringa–Kohn–Rostoker scheme can handle the finite-size slabs of two-dimensional PCs [23], and these methods may be extended to calculate LDF in three-dimensional PCs or two-dimensional PCs with finite-size slabs.

5. Conclusions

In summary, we investigated the lifetime distribution functions of SE from line antennas in two-dimensional finite-size 12-fold quasi-periodic PCs and found that SE is effectively controlled in different areas using quasi-periodic phononic crystals. SE in the central area, near the photonic band edge, and in the photonic gap can effectively be inhibited, while SE can be enhanced in the photonic band. However, SE in the edge area of quasi-periodic PCs, near the band edge, can apparently be enhanced.

Acknowledgment

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